

# The Coevolution of Individual Economic Characteristics and Socioeconomic Networks

Dietrich Stauffer<sup>1</sup>, Martin Hohnisch<sup>2,3</sup> and Sabine Pittnauer<sup>2</sup>

<sup>1</sup> Institute for Theoretical Physics, Cologne University, D-50923 Köln, Eu-  
roland

<sup>2</sup> Research Group Hildenbrand, Department of Economics, University of  
Bonn, Lennéstr. 37, D-53113 Bonn, Germany

<sup>3</sup> Department of Mathematics, University of Bielefeld, D-33501 Bielefeld,  
Germany

e-mail: stauffer@thp.uni-koeln.de; Martin.Hohnisch@wiwi.uni-bonn.de;  
Sabine.Pittnauer@wiwi.uni-bonn.de

**Abstract:** The opinion dynamics of economic agents is modeled with the link structure influenced by the resulting opinions: Links between people of nearly the same opinion are more stable than those between people of vastly different opinions. A simple scaling law describes the number of surviving final opinion as a function of the numbers of agents and of possible opinions.

## 1 Introduction

Local interaction structures, embodied in models of socioeconomic networks, have become increasingly recognized in economics as an extension of global interaction mechanisms. In this literature, economic networks are usually taken as exogenous, say a square lattice or a more complex graph. In this note we make an attempt to make the structure of links between agents endogenous, dependent on the degree of “similarity” between each pair of them. The point of departure is a random graph structure modified by the assumption that links associated with a node, i.e. an economic agent, are

not entirely random but influenced by the characteristics of other agents in the neighbourhood of that particular agent.

Simultaneously, we model the evolution of the characteristics themselves as a non-strategic social adaptation process. For concreteness, we call the characteristics simply “opinions”. While this is not the standard terminology in economic literature, the reader will recognize how the principle described extends to particular characteristics structures.

Our suggested process of opinion dynamics is based on previous work by [1]. Unlike in previous research, by the specification of endogenous links the network becomes dynamic, influencing the opinion dynamics and being influenced by it simultaneously.

Our investigation is based on computer simulation techniques using random numbers. Such an approach has a long history in economics [2].

## 2 Model

Our model uses simulation techniques known from the “sociophysics” literature of opinion dynamics [4] and Erdős-Rényi networks [5]. Each of  $N$  agents (we used  $N = 10^2, 10^3, 10^4$ ) can have one of  $Q$  opinions ( $10 \leq Q \leq 10^4$ ). The opinion of agent  $i$  is represented by the variable  $S_i$  taking values in a finite subset of  $[0, 1]$  consisting of the numbers  $n/Q$  with a natural number  $n \leq Q$ . At the outset, to each agent there is associated a reference group generated by a repeated random selection of agents (repeated ten times in our simulations). The generated link structure is assumed to be one-sided, i.e. if  $b$  is a reference person of  $a$  then not necessarily vice versa. For example, they may represent relations between agents and their superiors.

As in the model of Deffuant et al [1, 6], at every iteration each randomly selected agent  $i$  discusses successively with the agents in its reference group. In each instance, the two compare their opinions  $S_i$  and  $S_j$ . If their opinion

difference  $|S_i - S_j|$  is larger than a fixed confidence interval  $L$  (mostly between  $Q/20$  and  $Q/2$ ), they ignore each other's opinion; otherwise the two opinions move towards each other by an amount  $|S_i - S_j|/\sqrt{10}$ , rounded to the nearest integer value. (If their opinions agree, nothing changes; if their opinions differ by only  $\pm 1/Q$ , one of the two agents, randomly selected, adopts the opinion of the other.) This discretization of opinions to  $S = n/Q$  with natural numbers  $n$  between 1 and  $Q$ , instead of continuous numbers  $S$  between 0 and 1, is taken from [7] to improve computational efficiency; for the same reason we did not use the alternative model [8] where each agent looks at all  $N$  agents instead of only ten of them.

In addition, we allow for noise representing the random influence of the environment (good or bad economic news) [9], in addition to the opinion dynamics in the reference group. With probability 1/2, each agent shifts its opinion randomly up or down by  $\pm 1/Q$  (but stays within the interval from 0 to 1).

The coupling between the existence of a link between two agents and their opinions is to our knowledge the new aspect of our model: large differences of opinion destroy a link. Thus at each iteration, before the above opinion dynamics starts, the reference group of each agent  $i$  is reviewed. The link to agent  $j$  in the reference group is kept with a probability  $(p/Q)/|S_i - S_j|$  (if  $S_i = S_j$  the link is kept with probability 1), where  $p = 1/10, 1/2, 1$  was simulated ( $p = 1$  in all our figures). If a link is destroyed, another bond is selected randomly; if the two opinions of this new bond are far away from each other, this new bond will hardly survive the next iteration.

The simulations were continued either up to a fixed number  $10^4$  of iterations or until a fixed point is reached. (Each agent is treated on average once at each iteration; the number  $t$  of iterations thus measures the time.) We define a fixed point as a situation when without noise for ten consecutive iterations no opinion changed. With noise a fixed point is defined as a

situation where due to the opinion dynamics, ignoring the noise, during one iteration no opinion changed. The opinions at this fixed point are called final and are analyzed, with previous opinions ignored. Under conditions where noise prevents a fixed point to be reached within  $10^6$  iterations, we make only  $10^4$  iterations and average over all fluctuating opinion distributions in the second half of the simulations,  $5,000 < t \leq 10,000$ .

Without noise we always found fixed points; with noise depending on parameters we found fixed points, or we found opinions fluctuating about some stationary distributions. In most cases we averaged over 1000 samples to get smoother statistics; the Fortran program of about 140 lines is available as deffuant19.f from [stauffer@thp.uni-koeln.de](mailto:stauffer@thp.uni-koeln.de), as well as some figures mentioned but not shown below.

## 3 Results

### 3.1 No noise

Figure 1 shows that for large confidence intervals  $L$  spanning more than half of the possible opinion space a consensus is achieved: only one final opinion survives. For smaller  $L$  more than one opinion can survive; we always found a fixed point. The statistical fluctuations are barely visible, as was shown by another simulation using the same parameters but different random numbers. The deviations from the smooth curve  $Q/L$  shown as a dashed line in Figure 1 are thus systematic; only for  $2 \leq L \leq 50$  in the smooth left part the simulated results are proportional to  $Q/L$ . Similar data were obtained for different parameters  $N, Q, p$ , and also for the case without bonds where at each iteration each agent selects randomly one other agent for discussion (not shown).

In physics since 40 years many quantities were fitted on scaling laws. Thus a function  $z = f(x, y)$  of two variables  $x, y$  often can be written (for very

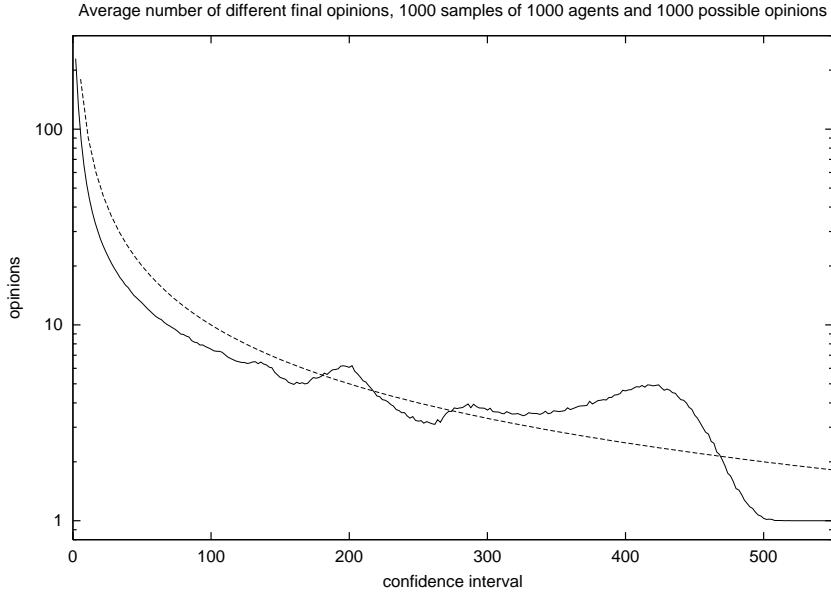


Figure 1: Semilogarithmic plot of the number of final opinions versus confidence interval  $L$ ; for  $L > Q/2$  (here = 500) a complete consensus for only one centrist opinion is seen.

small or very large  $x$  and  $y$ ) as a scaled variable  $z/x^a = F(y/x^b)$  given by a function  $F$  of only one scaled variable  $y/x^b$ , where  $a, b$  are free parameters often called critical exponents. In our case these exponents both are one, and the number  $M$  of surviving opinions, scaled by  $Q$ , is a function of the scaled variable  $N/Q$ . Figure 2 shows this scaling function in the form of  $M/(Q-1)$  versus  $Q/N$  for two drastically different system sizes  $N = 100$  and  $1000$  at  $L = 1$ : The two sets of data nicely overlap. For large  $Q/N$  most opinions have no adherents, nearly  $N$  different opinions have one adherent each, and very few opinions have two or more adherents. Thus only  $M = N$  opinions survive in this limit, and the scaled variable  $M/Q$  equals  $1/(Q/N)$  as shown by the straight line with downward slope in Figure 2. In the opposite limit of small  $Q/N$ , few agents initially share an opinion, the small confidence interval  $L = 1$  allows many different final opinions separated by more than

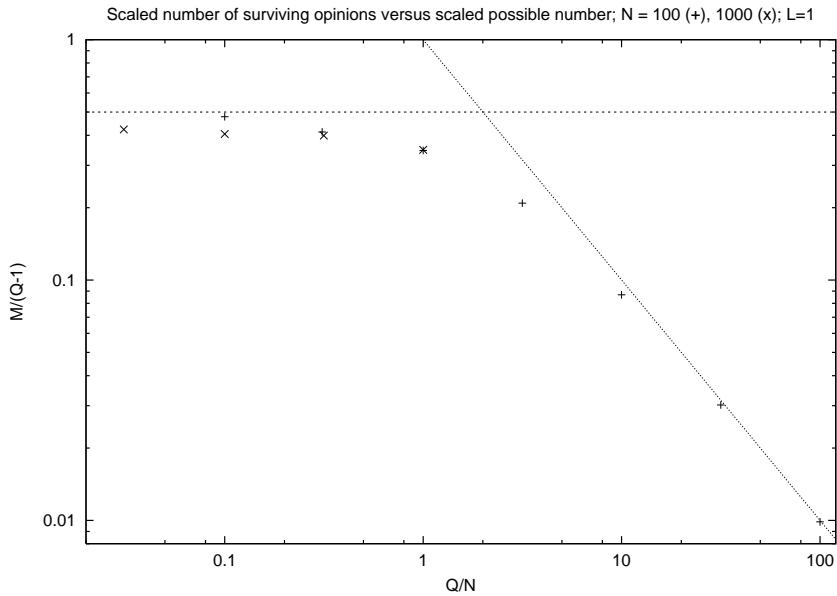


Figure 2: Double-logarithmic scaling plot. Different parameters  $N = 100$  and  $1000$  lead to the same curve.

$L$ , and thus the number  $M$  of final opinions is proportional to  $Q$ :  $M/Q = \text{const}$  in the left part of Figure 2, as indicated by the horizontal line. The simplicity of this scaling law explains that it is similar to the one found [7] in a different model of fixed links on a Barabási-Albert network. (We divided  $M$  by  $Q-1$  instead of by  $Q$  since for  $Q = 2$  a complete consensus  $M = 1$  was found; for the large  $Q \geq 10$  used here the difference hardly matters.) For  $L > 1$  a new parameter  $L/Q$  would have to be used, and the scaling would have been more complicated.

Figure 3 shows the dynamics until a fixed point is reached, at  $Q = N = 1000$ . For large  $L$  a complete consensus is reached as shown in Figure 1, and this case therefore is less interesting now. For small confidence intervals  $L$  we see in Figure 3 that the average difference of opinions with the ten agents in the reference group decreases exponentially with time  $t$ . The ten simulated samples give nearly the same results. For intermediate  $L = 150$

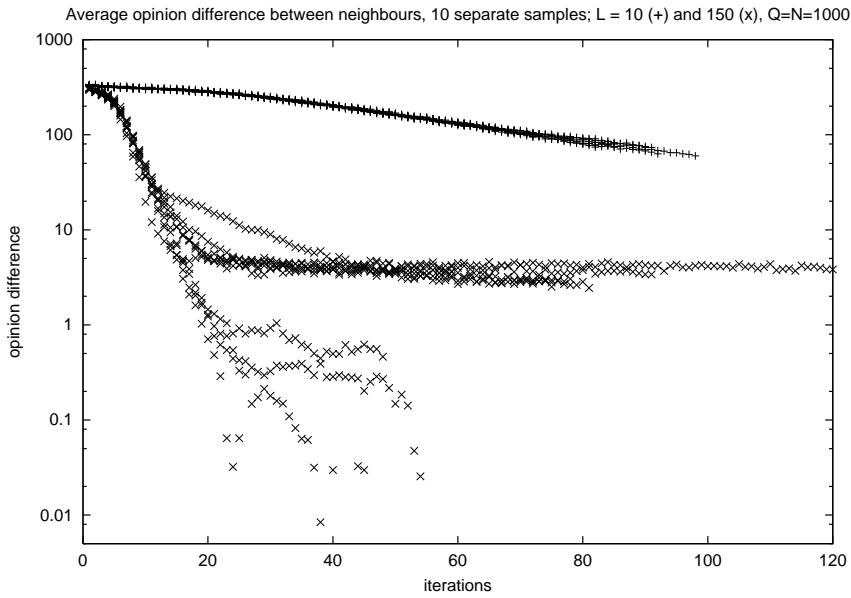


Figure 3: Semilogarithmic plot of the average opinion differences within a link, versus time. The upper data have a low confidence interval  $L = 10$ , the lower data an intermediate  $L = 150$ , for  $Q = 1000$ . Ten different simulations are shown separately (i.e. not averaged over). The opinions are multiplied here by  $Q$  and thus vary from 1 to 1000.

where on average about five opinions survive, we have drastic changes from sample to sample even though they differ only by the random numbers used: Sometimes the opinion differences reach a constant plateau, and sometimes they nearly vanish. In spite of its simplicity the model thus indicates that one cannot always predict that for two linked agents the opinions get close; one can only predict that the opinions for two linked agents get closer than they were at the beginning.

### 3.2 With noise

The constant noise disturbs the agreements which would have been found without noise, and thus the number  $M$  of final opinions in Figure 4 is much

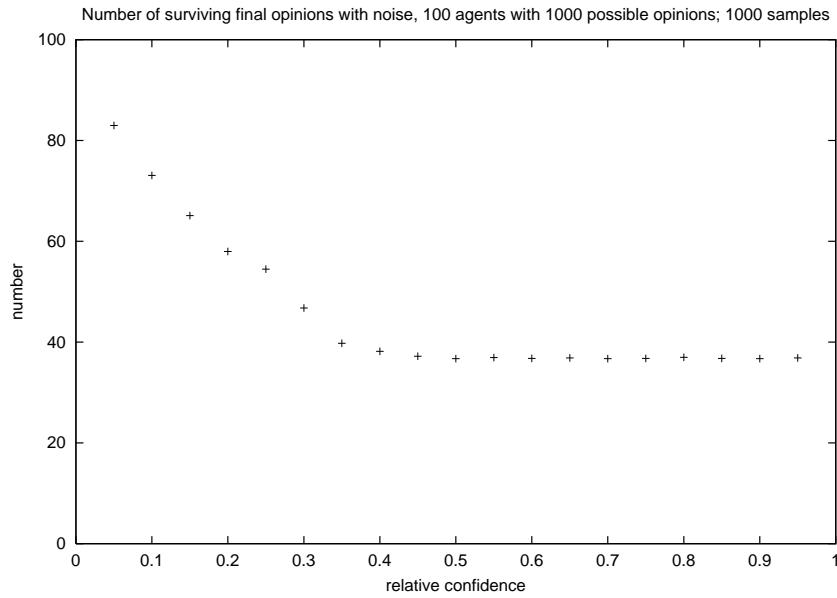


Figure 4: Linear plot of  $M$  versus  $L$  for strong noise;  $N = 100$ ,  $Q = 1000$ .

larger than without noise: Instead of only one surviving opinion for large confidence intervals  $L > Q/2$  we found on average 36.8 final opinions. For small  $L$  the number of surviving opinions is higher, as in Figure 1. The parameters  $N = 100$ ,  $Q = 1000$  were chosen such that always a fixed point was found.

With  $N = Q = 1000$  instead, no fixed point was found up to  $t = 10^6$ , and therefore we could look at the stationary distribution of the lifetimes for each link. We see in Figure 5 that there are many links with a lifetime of only one iteration; the number of observed lifetimes decays exponentially with increasing lifetimes, until for lifetimes of order  $10^2$  a plateau is reached, orders of magnitude below the maximum for unit lifetime.

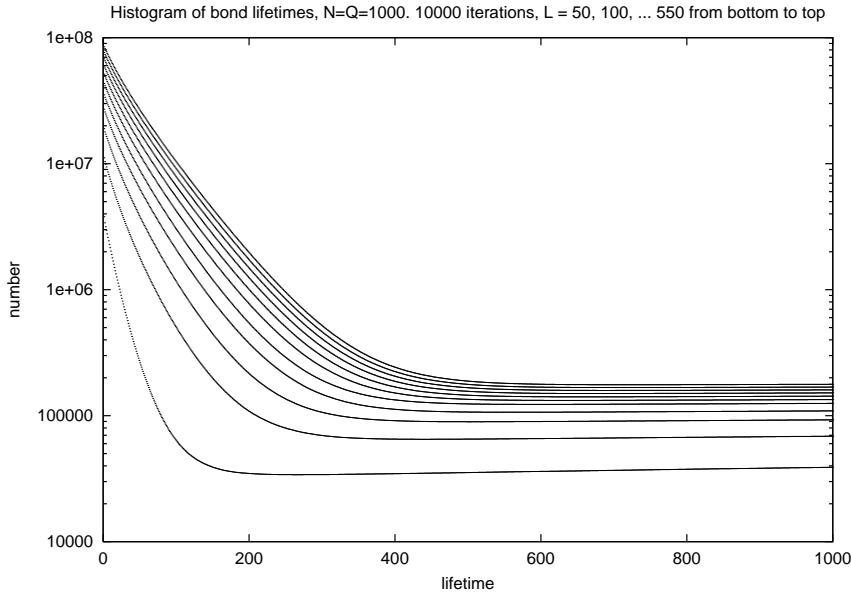


Figure 5: Semi-logarithmic histogram of lifetimes of links. Since each of the 1000 agents in each of the 1000 samples contributes ten differences, the data are much more smooth (“self-averaging”) than those in the earlier figures where e.g. in Figure 1 the whole sample gave only one number  $M$ .

## 4 Summary

In this model of opinion dynamics, for large enough confidence intervals  $L > Q/2$  everybody finally agrees with one centrist opinion, while initially the opinions were distributed randomly. In the case no such consensus is reached, the number  $M$  of surviving opinions obeys a simple scaling law,  $M = Q \cdot F(N/Q)$ , as a function of the number  $N$  of agents and the number  $Q$  of possible opinions, for large  $N$  and  $Q$ . The consensus might correspond in reality to market bubbles, like for information technology stocks before spring 2000, or for tulips centuries ago. Our evolutionary process includes self-organisation of the network of links between agents, depending on their opinions, and influencing in turn their opinions. Improvements like inclusion of value judgments between good and bad opinions, or influence of punctual

events on the opinions, are in preparation.

MH gratefully acknowledges financial support from DFG grant TR 120/12-1, DS that from COST P10 of the European Science Foundation.

## References

- [1] G. Deffuant, D. Neau, F. Amblard and G. Weisbuch, *Adv. Complex Syst.* 3, 87 (2000); G. Weisbuch, G. Deffuant, F. Amblard, and J.-P. Nadal, *Complexity* 7, 55 (2002); Deffuant, G., Amblard, F., Weisbuch G. and Faure, T., *Journal of Artificial Societies and Social Simulation* 5, issue 4, paper 1 ([jasss.soc.surrey.ac.uk](http://jasss.soc.surrey.ac.uk)) (2002).
- [2] G.J. Stigler, *Journal of Business* 37, 117 (1964).
- [3] R. Cont and J.P. Bouchaud, *Macroeconomic Dynamics* 4, 170 (2000).
- [4] R. Axelrod, *J. Conflict Resolut.* 41, 203 (1997).
- [5] S. Bornholdt and H.G. Schuster (Eds.), *Handbook of Graphs and Networks - From the Genome to the Internet*. Wiley-VCH, Weinheim 2003.
- [6] E. Ben-Naim, P. Krapivsky and S. Redner, *Physica D* 183, 190 (2003).
- [7] D. Stauffer, A.O. Sousa and C. Schulze, preprint for *Journal of Artificial Societies and Social Simulation*.
- [8] R. Hegselmann and U. Krause, *Journal of Artificial Societies and Social Simulation* 5, issue 3, paper 2 ([jasss.soc.surrey.ac.uk](http://jasss.soc.surrey.ac.uk)) (2002) and *Physica A*, in press (2004); U. Krause, p.37 in *Modellierung und Simulation von Dynamiken mit vielen interagierenden Akteuren*, edited by U. Krause and M. Stöckler, Bremen University, Jan. 1997.
- [9] J. Hołyst, priv. comm.; D. Stauffer and H. Meyer-Ortmanns, *Int. J. Mod. Phys. C* 15, issue 2 (2004).